Random Lattice Triangulations

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Abstract

A lattice triangulation is a triangulation of the set of integer points in an nxm rectangle (or, more generally, a polygon) in the plane. Lattice triangulations have been studied in geometry, combinatorics, algebraic geometry and statistical physics. This talk discusses structural and algorithmic properties of random lattice triangulations. We first generalize the model by assigning weight $c^{l(T)}$ to triangulation T, where $l(T)$ is the total length of the edges of T and c is a parameter. Thus when $c = 1$ the weights are uniform, while $c < 1$ and $c > 1$ favor triangulations with shorter and longer edges respectively. We conjecture on the basis of experiments that this model exhibits a phase transition at $c = 1$: i.e., for all $c < 1$ correlations between edges decay with distance, and the natural Markov chain Monte Carlo algorithm ("edge-flip dynamics") for sampling triangulations has mixing time polynomial in n, m; while for $c > 1$ there are long-range correlations and the mixing time is exponential. We give various pieces of evidence for this conjecture, namely decay of correlations and polynomial mixing time for sufficiently small c, and exponential mixing time for all $c > 1$. We also prove the analogous conjecture for the case of "thin" rectangles, where m is any fixed constant.

Joint work with *Pietro Caputo, Fabio Martinelli* and *Alexandre Stauffer*